Even though a human can trivially decide an arbitrarily long bitstring of 1s is not random, Theorem 3 shows is an impossible task for a generalized algorithm. Only a specific algorithm, such as exemplified in Theorem 2, can do so.

This conclusion is a bit counter-intuitive, since it means that without domain knowledge, an algorithm given an extremely long sequence of 1s would be unsure whether the sequence is completely random. When asked to predict the next digit, the algorithm can only give an equal weighting to 0 and 1.

Proving the Derivative of $\sin(x)$ Using the Pythagorean Theorem and the Unit Circle

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DOI: 10.33014/issn.2640-5652.3.1.bartlett.1

The derivative of $\sin(x)$ (where x is measured in radians) is given in standard calculus as $\cos(x)$. The proof for this is usually based on a limit: $\lim_{q \to 0} \frac{\sin(q)}{q} = 1$. The proof, put simply, is:

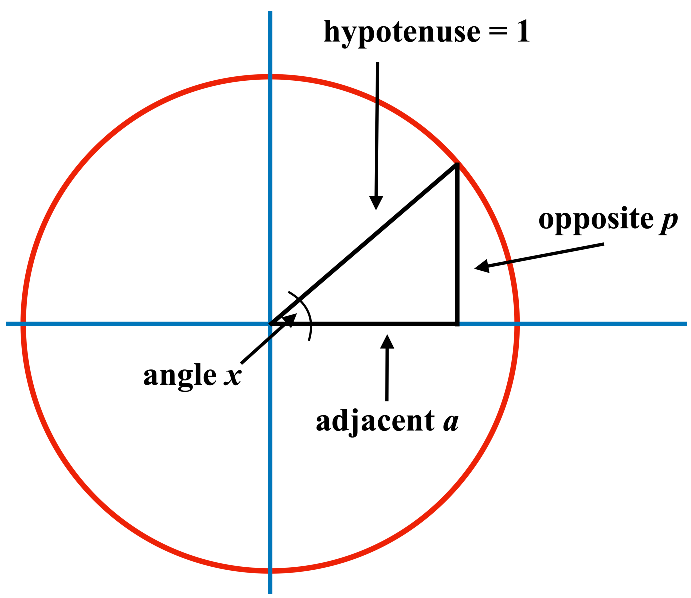
$$y = \sin(x) \tag{1}$$
$$y + \mathrm{d}y = \sin(x + \mathrm{d}x) \tag{2}$$
$$\mathrm{d}y = \sin(x + \mathrm{d}x) - \sin(x) \tag{3}$$
$$\mathrm{d}y = \sin(x)\cos(\mathrm{d}x) + \cos(x)\sin(\mathrm{d}x) - \sin(x) \tag{4}$$
$$\mathrm{d}y = \sin(x) + \cos(x)\sin(\mathrm{d}x) - \sin(x) \tag{5}$$
$$\mathrm{d}y = \cos(x)\sin(\mathrm{d}x) \tag{6}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(x)\frac{\sin(\mathrm{d}x)}{\mathrm{d}x} \tag{7}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(x) \tag{8}$$

While there is nothing wrong with the proof per se, I have always found it unsatisfying, utilizing trigonometry identities few students remember. Additionally, it is usually accompanied with an explanation of the limit of $\frac{\sin x}{x}$ that is hard for students to decipher. Therefore, this paper endeavors to provide a more straightforward proof based on more basic mathematical assertions, founded on the Pythagorean theorem and the unit circle. It doesn't remove the given limit in its entirety, but rather gives more straightforward, calculus-oriented reasoning for doing a similar operation. It is debatable how much different it is *in kind* from the standard proof, but in any case I think it is a more straightforward, interesting, and instructive way of looking at it for students. It shows (a) the power of calculus, (b) the power of differential thinking, and (c) how discoveries can be made from basic principles.

Basic Assumptions

This proof will be analyzing triangles drawn on the unit circle. On a unit circle, the hypotenuse will always be 1. Figure 1 shows the general setup. x will be the angle measured in radians, a will be the adjacent, and p will be the opposite.

Figure 1: A Triangle Inscribed Onto a Unit Circle



The Pythagorean theorem gives the following:

$$a^2 + p^2 = 1 \tag{9}$$
$$p^2 = 1 - a^2 \tag{10}$$
$$a^2 = 1 - p^2 \tag{11}$$
$$\tag{12}$$

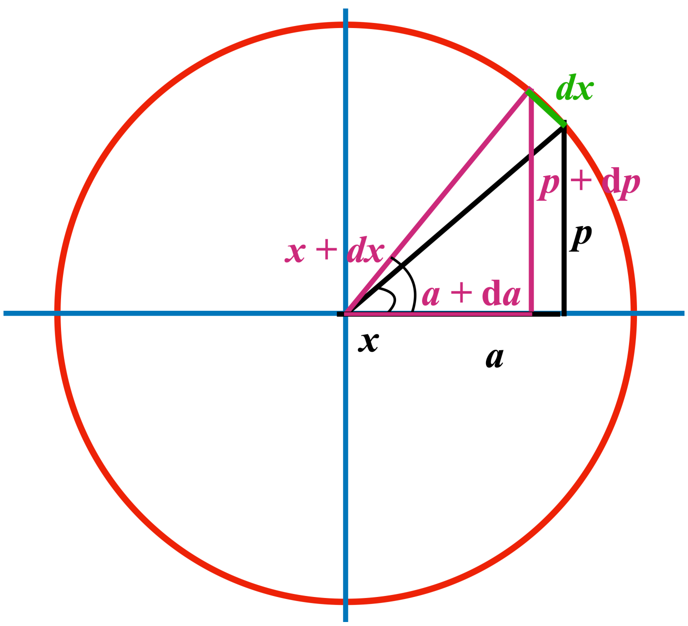
Since the hypotenuse is 1, $\sin(x) = p$ and $\cos(x) = a$. The derivative of $\sin(x)$ with respect to x, therefore, will be $\frac{\mathrm{d}p}{\mathrm{d}x}$. Therefore, the proof will be successful if it can demonstrate the following equivalency:

$$\frac{\mathrm{d}p}{\mathrm{d}x} = a \tag{13}$$

Differential Analysis

Taking Figure 1 and budging the angle by $\mathrm{d}x$ yields the picture shown in Figure 2.

Figure 2: Changes in the Triangle Based on $\mathrm{d}x$



A few important notes on Figure 2:

1. All changes are being expressed as *adding* differentials, even if the differential itself is negative. This is why $a+\mathrm{d}a$ in the graph is shorter than a on its own.
2. Since this is the unit circle, the angle change *is identical* to the circumference change (since the radius is 1, then the circumference is 2π, the number of radians in a circle).
3. Since the change is infinitesimal, and this is a smooth and continuous figure, then the change on the differential is *linear*. In other words, the picture is zoomed in enough that the arc joining the two triangles can be treated as if it were a straight line.[1]

Because of this last point, the length of $\mathrm{d}x$ can be determined using the distance formula, where the horizontal and vertical changes are simply given by $\mathrm{d}a$ and $\mathrm{d}p$:

$$\mathrm{d}x = \sqrt{\mathrm{d}p^2 + \mathrm{d}a^2} \tag{14}$$

Finally, the differential of (9) can be taken to come up with:

$$a^2 + p^2 = 1$$

$$2a\,\mathrm{d}a + 2p\,\mathrm{d}p = 0 \tag{15}$$

$$a\,\mathrm{d}a + p\,\mathrm{d}p = 0 \tag{16}$$

$$a\,\mathrm{d}a = -p\,\mathrm{d}p \tag{17}$$

$$\mathrm{d}a = -\frac{p}{a}\,\mathrm{d}p \tag{18}$$

Making the Proof

Starting with (14), substitutions and simplifications can be made as follows:

$$\mathrm{d}x = \sqrt{\mathrm{d}p^2 + \mathrm{d}a^2} \tag{19}$$

$$= \sqrt{\mathrm{d}p^2 + \left(-\frac{p}{a}\,\mathrm{d}p\right)^2} \tag{20}$$

$$= \sqrt{\mathrm{d}p^2 + \frac{p^2}{a^2}\,\mathrm{d}p^2} \tag{21}$$

$$= \sqrt{\mathrm{d}p^2 + \frac{1-a^2}{a^2}\,\mathrm{d}p^2} \tag{22}$$

$$= \sqrt{\mathrm{d}p^2 + \frac{\mathrm{d}p^2}{a^2} - \mathrm{d}p^2} \tag{23}$$

$$= \sqrt{\frac{\mathrm{d}p^2}{a^2}} \tag{24}$$

$$\mathrm{d}x = \frac{\mathrm{d}p}{a} \tag{25}$$

Note that (25) could also have been negative. Inspection of Figure 2 shows that $\mathrm{d}p$ will always have the same sign as a (increasing until a is zero, then decreasing while a is negative). Therefore, choosing the positive square root is the valid choice.

As stated at the beginning, the goal is to figure out an alternative reading of $\frac{\mathrm{d}p}{\mathrm{d}x}$. Using (25), this can be simplified as follows:

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mathrm{d}p}{\frac{\mathrm{d}p}{a}} = \frac{\mathrm{d}p}{1}\frac{a}{\mathrm{d}p} = a \tag{26}$$

As shown in (13), this proves that the derivative of $\sin(x)$ is indeed $\cos(x)$.

[1]Note that this is basically equivalent to the limit $\lim_{q\to 0} \frac{\sin(q)}{q}$, but stated in a more straightforward way that is repeatedly in calculus thinking.

What is significant about this proof is that it relies entirely on the basics—the Pythagorean theorem, the unit circle, the definition of sine and cosine, the definition of the radian measure of an angle, the distance formula, and the power rule.

A Response to Clunn's Axioms of Morality

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DOI: 10.33014/issn.2640-5652.3.1.miller.1

This article offers a brief critique of Clunn's foundationalism which grounds moral decision making in what he calls the three fundamental axioms of existence, consciousness, and identity (Clunn, 2019). It shows how his commitment to neo-Platonism, or possibly pantheism, creates at least three incoherencies wherein *a priori* is *a posteriori*, individuality is an illusion, and objective morality is subjective. For Clunn's moral philosophy to offer practical value, these internal conflicts must be resolved.

Introduction

In his article, *Axioms of Morality*, Clunn argues that morality is an *a priori* truth that is objectively known to every person. He believes that the fundamental axioms of existence, consciousness and identity make life itself the ultimate objective standard for each person's subjective moral choices. Therefore, as a general rule, any moral choice which benefits life in general is a moral good. Any moral choice which hurts life in general is a moral evil. Clunn rejects selfishness and utilitarianism as viable methods for choosing what is good. Instead, he argues that our individual choices must be guided by what he considers the four cardinal virtues defined by history: justice, prudence, temperance and courage.

Finding an objective ground for moral good is a daunting task for any philosopher. And while Clunn's three axioms are important, the grounding for his overarching moral philosophy is problematic. For Clunn, every person shares in the same *a priori* universal consciousness which is a nonreductive emergent property of the biological structures that define humanness. If it is true that existence, consciousness and identity exist *a priori* to human life in some form of neo-Platonic realm—or possibly in a pantheistic universe—at least three internal conflicts arise.

Conflict #1: *A Priori* is *A Posteriori*

Clunn leans heavily on Ayn Rand for defining his axioms but departs from Rand who taught that consciousness and morality are *a posteriori*. This distinction is critical for Clunn as he hopes to sustain his commitment to both objective morality and free will. He writes, "At the end of the day, morality is about free will, choices, and decisions. These things all exist within our consciousness (45)." Here is where the incoherence first manifests. By definition, *a priori* means that morality must exist independent of any person's experience. Yet, Clunn also presumes that morality exists through the exercise of one's free will. Given these claims, morality must also be *a posteriori* because it depends upon how each individual person exercises their free will. Clunn's presupposition of *a priori* morality may be preserved if he assumes free will is also an expression of the *a priori* universal consciousness. However, this assumption leads to a second conflict for how Clunn defines identity.

Conflict #2: Individuality is an Illusion

For Clunn, consciousness is not a property of personhood, but an emergent property of the physical realm that existed before any individual. That is to say, all humans share in the one nonreductive *a priori* universal consciousness. At the same time, Clunn argues that the term "I" is an expression of rational thought which establishes one's specific identity within the universal consciousnesses. But even if "I" establishes my personal existence, it remains an existence only within the larger axiom of existence. It seems to follow from Clunn's own definitions that the perception of individuality, and by extension free choice, is only an illusion. This creates at least one significant internal conflicts for Clunn's axioms.

Clunn argues that the consequences of our decisions are experienced only within the realm of personal consciousness, which no other person can observe. In contrast, Clunn says that we can observe existence. Yet, for Clunn, morality does not manifest in the axiom of existence. However, if each person's consciousness is a shared *a priori* reality, how is it beyond my powers of observation? If I can have awareness of my own consciousness, and that consciousness is tied to the universal, then by definition I must also have access to understanding the consciousness of others because they too are tied to the same universal axiom. Even more, if consciousness is an emergent property of existence, how does it remain independent of existence as it relates to morality? This incoherence leads to